

## **Damage index-based restoring force model for seismic damaged compression-bending RC members**

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### **ABSTRACT**

A restoring force model of Seismic-Damaged Compression-Bending(SDCB) RC members based on the damage index is proposed in this paper. Strength and stiffness reduction coefficients is introduced to solve the problem of the relationship between the damage degree and the mechanical property degradation. The restoring force model of SDCB RC members based on ModIMK material model is constructed basing on the cyclic strength degradation reduction method with hysteretic energy consumption. The relationship between member damage index and strength and stiffness reduction coefficients is established by using data fitting regression method to quantify the relationship between RC member damage and performance degradation of SDCB RC members. Considering the stochastic nature of RC members and earthquakes, the probability distribution curves of strength and stiffness reduction coefficients of SDCB RC members are proposed.

### **1. INTRODUCTION**

Reinforced concrete material is the most rapidly developing building material in the last century. The research on its restoring force model has achieved fruitful results. The seismic damage assessment of RC members has authoritative theoretical basis and rich experimental and practical experience. Currently, the most widely used restoring force model of reinforced concrete is ModIMK material model<sup>[1]</sup>. ModIMK material model considers the strength degradation after the ultimate strength of the member and introduces the residual strength to represent the residual strength that remains when the member reaches the specified degradation limit. The cyclic degradation mode of accelerated reloading of ModIMK model can accurately simulate the degradation mode of relevant parameters of RC members<sup>[1-3]</sup>. However, there are still many technical problems in seismic performance evaluation of damaged RC structures. The

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relationship between seismic damage level and structural performance degradation of RC members is one of the technical difficulties in seismic engineering.

Structural seismic damage assessment is of great significance for structural seismic assessment. It has been applied in the field of concrete damage theory for decades<sup>[4-6]</sup>. The two parameter seismic damage model proposed by Park and Ang in 1985 considering the properties of member deformation and energy consumption has been recognized by the engineering and academic circles<sup>[6]</sup>, and the improved Park and Ang two parameter damage model proposed by Kunnath<sup>[7]</sup>.

A skeleton curve model for seismic damaged compression bending (SDCB) RC element is proposed. Using RC experimental data in PEER structural performance database, the empirical relationship between performance reduction and damage index is established by regression analysis.

## 2. Skeleton curve model of SDCB RC members based on damage index

### 2.1 Skeleton curve model of SDCB RC members

The form of the skeleton curve of the SDCB RC member is assumed to be the same as that of the undamaged RC member, as showed in figure 1. To determine this skeleton curves of SDCB members based on the ModIMK model, the following parameters need to be determined: yield strength ( $M_{yd}$ ), elastic stiffness ( $K_{ed}$ ), ultimate strength ( $M_{ud}$ ), ultimate displacement ( $\theta_{ud}$ ), softening stiffness coefficient ( $\alpha_{cd}$ ), residual strength ( $M_{rd}$ ), and damage displacement ( $\theta_{cd}$ ).

In this paper, the following assumptions are made based on the existing research base for the skeleton curves of the seismic damage RC members based on the ModIMK material model.

- (1) The ultimate displacement of the member remains unchanged, i.e.  $\theta_u = \theta_{ud}$ .
- (2) The residual strength of the member remains constant, i.e.  $M_r = M_{rd}$ .
- (3) The damage displacement of the member remains constant, i.e.  $\theta_c = \theta_{cd}$ .
- (4) The softening stiffness factor of the member is the same as that of the undamaged member, i.e.  $\alpha_c = \alpha_{cd}$ .

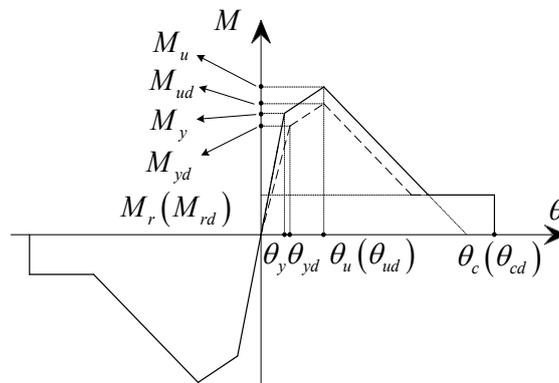


Fig. 1 Backbone curve for SDCB RC members

Based on the above assumptions, three parameters are needed to determine the skeleton curve of SDCB RC components, which are yield strength ( $M_{yd}$ ), elastic stiffness ( $K_{ed}$ ), and ultimate strength ( $M_{ud}$ ). The yield strength can be determined using the hysteretic energy dissipation based cyclic strength degradation reduction method by introducing the cyclic degradation factor  $\beta_i$ .  $\beta_i$  represents the degradation factor of the RC member under cyclic loading at the  $i$ th cycle<sup>[1]</sup>.

$$\beta_i = \left( \frac{E_i}{E_t - \sum_{j=1}^{i-1} E_j} \right)^c \quad (1)$$

where  $E_i$  represents the energy dissipated by the member in the first hysteresis cycle,  $\sum E_j$  represents the energy dissipated by the member during the previous loading history,  $E_t$  represents the hysteresis dissipation capacity of the member, usually denoted as  $E_t = \gamma F_y x_y$ , the parameter  $\gamma$  is related to the degradation model,  $c$  is the degradation rate index, Rahnama and Krawinkler<sup>[9]</sup> recommended a reasonable range of values for  $c$  from 1.0 to 2.0. When the displacement time-history consists of constant amplitude cycles,  $c$  can take the value 1, which indicates an almost constant degradation rate. When  $c$  takes the value 2, indicates a slower degradation rate in the early cycles and a faster degradation rate in the later ones.

According to the definition of cyclic strength degradation based on hysteresis energy dissipation, the following relationship exists between the yield strengths  $F_{i-1}$  and  $F_i$  before and after the  $i$ th hysteresis cycle.

$$F_i = (1 - \beta_i) F_{i-1} \quad (2)$$

The relationship between the yield strength of the member after cyclic loading and the initial yield strength can be expressed as :

$$F_{yd} = F_y \cdot \prod (1 - \beta_i) \quad (3)$$

The cyclic strength degradation based on hysteretic energy dissipation is more representative of the strength reduction effect due to cyclic loading than the strength loss theory. The strength degradation factor  $\alpha_F$  is introduced as:

$$\alpha_F = \prod (1 - \beta_i) \quad (4)$$

The yield strength after experiencing cyclic loading can be expressed as:

$$F_{yd} = \alpha_F \cdot F_y \quad (5)$$

The shear model can be converted to the bending moment model.

$$M_{yd} = \alpha_F \cdot M_y \quad (6)$$

The yield displacement can be determined using the assumption as Equation (7).

$$\theta_{yd} = \begin{cases} \theta_y; & \Theta_b < \theta_y \\ \Theta_b; & \Theta_b \geq \theta_y \end{cases} \quad (7)$$

Where both  $\Theta_b$  and  $X_b$  represent the average value of the absolute value of each extreme point in the displacement time course curve of the member under the historical load. The elastic stiffness of the damaged member can be expressed as:

$$K_{ed} = \begin{cases} M_{yd} / \theta_y; & \Theta_b < \theta_y \\ M_{yd} / \Theta_b; & \Theta_b \geq \theta_y \end{cases} \quad (8)$$

The ultimate strength of the damaged member can be calculated using the following equation (9), assuming equal strength loss.

$$M_{ud} = M_u - (1 - \alpha_F) \cdot M_y \quad (9)$$

## 2.2 Damage model

Park and Ang proposed the most representative two-parameter earthquake damage model in 1985<sup>[7]</sup>. In this damage model, damage index is a linear combination of the maximum deformation and the accumulated hysteresis energy of the member. Kunnath<sup>[8]</sup> modified the Park-Ang damage model in the elastic phase of the damage index is greater than 0, and put forward the damage index calculation formula adapted to the bending members.

$$DI = \frac{\theta_m - \theta_{re}}{\theta_u - \theta_{re}} + \beta \frac{E_h}{M_y \theta_u} \quad (10)$$

where  $\theta_{re}$  is the recoverable angle of the member section when unloaded,  $\theta_m$  and  $\theta_u$  represent the maximum and ultimate angles of the member section respectively,  $E_h$  is the capacity consumed by the section,  $\beta$  is the energy dissipation factor of the member.

The key to determine the skeleton curve of the seismically damaged RC member is to determine the elastic stiffness and yield strength. The stiffness degradation factor ( $\alpha_K$ ) is introduced here to describe the elastic stiffness reduction of the damaged member.

$$\alpha_K = K_{ed} / K_e \quad (11)$$

## 2.2 Relationship between damage and performance reduction

The PEER structural performance database contains a rich collection of test data related to the mechanical performance study of RC members. In this paper, a total of 1447 groups of component damage index and component stiffness and strength reduction factors were collected. Non-linear regression was used to obtain the relationship between the damage index of the member and the strength and stiffness reduction coefficients.

$$\alpha_F = f_F(DI) \quad (12)$$

$$\alpha_K = f_K(DI) \quad (13)$$

Figure 2 shows the collected damage indexes and strength reduction coefficients. The distribution of sample points in Figure 2 shows a clear correlation between the damage index and the strength reduction factor of RC members. The lower and upper bound estimates of the strength reduction coefficients of the seismic damaged members are defined to describe the relationship between the reduction coefficients of the seismic damaged reinforced concrete members and the seismic damage of the members. The lower bound estimate means that 95% of the sample points have a strength reduction factor greater than the lower bound estimate. 95% of the sample points in Figure 5 are above the lower bound estimate curve; the upper bound estimate

means that 5% of the sample points have a strength reduction factor greater than the upper bound estimate. 5% of the sample points in Figure 5 are above the upper bound estimate curve. The equations for the mean estimate, lower and upper bound estimates of the strength reduction coefficients of the seismic damaged members are Eqs. (14), (15) and (16), respectively.

$$\alpha_{F,aver} = \frac{0.07}{0.08 + DI^{3.16}} + 0.125 \quad (14)$$

$$\alpha_{F,low} = \frac{0.07}{0.08 + DI^{2.5}} + 0.120 \quad (15)$$

$$\alpha_{F,up} = \frac{0.07}{0.08 + DI^6} + 0.125 \quad (16)$$

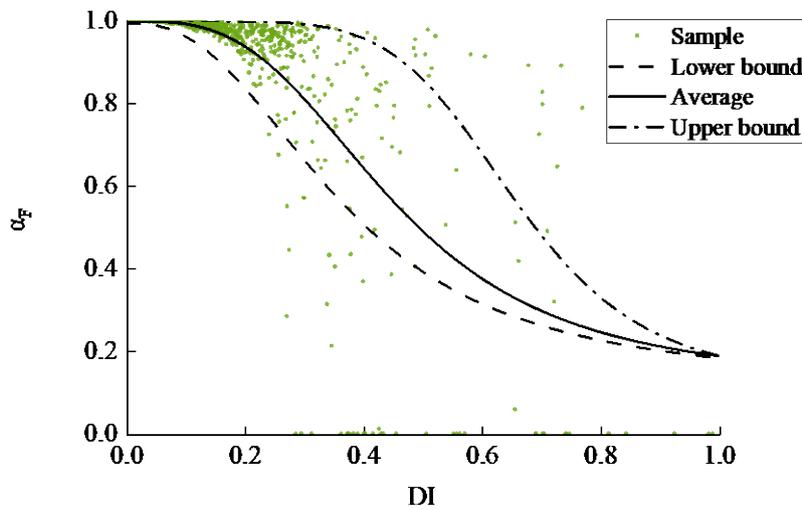


Fig. 2 Fitting curves for strength reduction factor

Figure 3 shows the collected damage indexes and stiffness reduction coefficients. There is a clear correlation between the damage index and the stiffness reduction factor. Equations (24), (25), and (26) are respectively the equations for the average estimate, lower bound estimate, and upper bound estimate of the stiffness reduction factor of RC members obtained by nonlinear regression.

$$\alpha_{K,aver} = \frac{1}{1 + 5.55DI - 0.51DI^2 + 15.15DI^3} \quad (17)$$

$$\alpha_{K,low} = \frac{1}{1 + 25DI - 0.51DI^2 + 15.15DI^3} \quad (18)$$

$$\alpha_{K,up} = \frac{1}{1 + 0.01DI - 1.11DI^2 + 15.15DI^3} \quad (19)$$

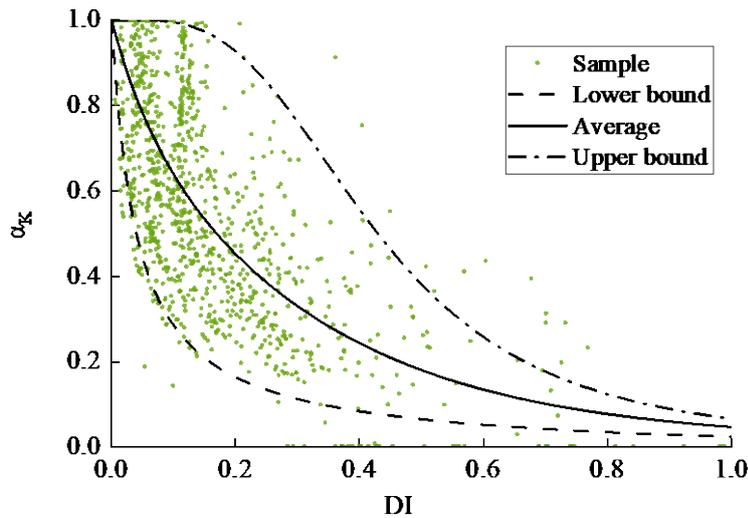


Fig. 3 Fitting curves for stiffness reduction factor

### 3. CONCLUSIONS

In this paper, the hysteresis rules of the seismic damage buckling members are the same as those of the intact members in determining the restoring force of the SDCB RC members, focusing on the skeleton curves of the seismic damage buckling RC members. The main research contents and conclusions are as follows.

(1) A skeleton curve model of SDCB RC members is presented in this paper. This model uses the principle of cyclic strength degradation based on hysteretic energy dissipation, and introduces the member strength and stiffness degradation factor to describe the performance degradation caused by member damage, which more accurately describes the cyclic performance degradation of the member.

(2) Using the cyclic tests of bending damaged RC members in the PEER database, the relationship between the damage index of the members and the strength and stiffness reduction coefficients of the seismic damaged members is proposed by statistical analysis. The quantified relationship between the damage level and the performance degradation of the members are presented.

(3) Considering the extremely complex mechanical properties of reinforced concrete, three possible forms of estimation were considered simultaneously, which are average estimation, lower bound estimation, and upper bound estimation. The cumulative probability distribution of the strength and stiffness discount factors of the SDCB RC members was proposed.

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